Generalized Particle Swarm Optimizers with Tracking Multiple Local Optima for Multimodal Functions Optimization

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Abstract

This paper presents a new variation of particle swarm optimization (PSO) algorithm called generalized particle swarm optimizer (GPSO). It extends the basic learning strategy of traditional PSO and exerts the swarms to significantly improve the group learning performance. In this scheme, a particle of PSO in each dimension does not only follow its own local optima, but also follows other superior particles' local optima with creditability. Based on our experimental verifications, the results suggest that GPSO delivers superior performance for multimodal functions optimization compared with the state-of-art PSO methods.

1. Introduction

Easy and simple problems can be well solved by traditional optimization methods using gradient information. For solving complex or non-derivative optimization problems, evolutionary computation techniques have been used considerably and further improved resulting in a set of modern heuristics tools like simulated annealing (SA), genetic algorithm (GA), ant colony optimization (ACO) and particle swarm optimization (PSO) over the past several years. These computational intelligence methods have demonstrated better characteristics in dealing with complex problems than conventional optimization techniques. PSO as a representative of these evolutionary algorithms has some appealing features including less parameters tuning and fast convergence rate. It also performs well in a wide variety of applications such as neural network learning, pattern recognition, and data mining, etc. [2].

Despite the appealing optimization characteristics offered by PSO, it is notorious of being easily trapped in local minima like other evolutionary computation techniques and results in the possibility of non-stability after continuously independent runs. In order to overcome these shortcomings, many researchers have presented various versions of modified PSOs. Veeramachaneni and Peram et al. presented a fitness-distance-ratio-based PSO algorithm with near neighbor interactions by using the ratio of the relative fitness and the distance of other particles [3]. Parsopoulos and Vrahatis [4] proposed a unified particle swarm optimizer that harnesses the local and global variant of PSO without imposing additional requirements in terms of function evaluations. Mendes and Kennedy et al [5] used all the neighbors of the particle instead of locally best and globally best positions to update the velocity. At the point of dimension learning, a cooperative approach to PSO is achieved by using multiple swarms to optimize different components of the solution vector cooperatively [6]. Likewise Liang et al. [7] presented a comprehensive learning particle swarm optimizer using a novel learning strategy. Moreover, by employing other searching techniques combined into PSO, hybrid improved PSOs are also investigated by researchers [8]. It is worth noting that combining other searching techniques into PSO may have improved the performance but it usually results in complicating the requirements of empirical parameters.

In this paper, we extend the basic model of PSO and present a generalized particle swarm optimizer (GPSO) with social behavior concept. To strength exploitation in evolutionary process, updating of particles in our proposed GPSO is based on tracking other local optima together with its own local optimum with credit coefficients. To balance exploration and exploitation, credit coefficients are varied dynamically. Intensive simulations were conducted and our proposed algorithm is compared with other six different PSOs under classical benchmark problems. GPSO delivers better performance than other PSOs over the experiments in terms of convergence and stability.

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2 Standard PSO

A particle in the searching space is characterized by two factors: position and velocity. The position and the velocity of the ith particle in the d-dimensional search space can be represented as $X_i = (x_{i,1}, x_{i,2}, \ldots, x_{i,d})$ and $V_i = (v_{i,1}, v_{i,2}, \ldots, v_{i,d})$, respectively. The ith particle has its own best position $P_i = (p_{i,1}, p_{i,2}, \ldots, p_{i,d})$, corresponding to the individual best objective value obtained so far at time t. The global best particle is denoted as g, which represents the best position found so far at time t, in the whole swarm. The new velocity of each particle is given by [1]:

$$v_{i,j}(t+1) = wv_{i,j}(t) + c_1 r_1 [p_{i,j} - x_{i,j}(t)] + c_2 r_2 [g_j - x_{i,j}(t)] \quad (1)$$

where c_1 and c_2 are constants named acceleration coefficients, usually $c_1 = c_2 = 1.49$; r_1 and r_2 are two independent random numbers uniformly distributed in the range $[0, 1]$. w is the inertia weight. Empirical studies show that the convergence performance can be greatly improved if w in the range $[0.4, 0.9]$ declines linearly along with the exploration proceeding. The updating equation is given by

$$w(t) = w_{\max} - t \times \frac{(w_{\max} - w_{\min})}{n_{\max}} \quad (2)$$

where $w_{\max}$ and $w_{\min}$ are the maximum and minimum inertia weights, respectively, t is the current iteration number, and $n_{\max}$ is the maximum number of iteration. The position of each particle is then updated in iteration according to the following equation

$$x_{i,j}(t+1) = x_{i,j}(t) + v_{i,j}(t+1) \quad (3)$$

Generally the value of each component in V_i can be clamped into the range $[v_{\min}, v_{\max}]$ in order to control the excessive roaming of particles outside the searching space. The search process repeats until the maximum number of iterations is reached or the stopping criterion is satisfied.

3 GPSO

3.1 Motivation

Social behavior may be considered to be a complex network, in which the relationships among individuals are time-varying, unstable and frail. Although it is difficult to emulate this behavior completely, this phenomenon can be modeled for optimizing problems in computational research. Metaphysically, each social swarm consists of many groups in which each individual is close to each other such as a family, a company and even a country. When there are benefits such as food and good opportunities, the close relationship will force one to share this information with one's neighborhoods or companions without considering internal competition. In addition, one's neighborhoods will not query the information and follow it for achieving the goal. However, when the whole group approaches the same common goal, the creditability among individuals will decline rapidly. This phenomenon has well been manifested in the world financial speculation market. There also exists a very strong basis in the field of micro-economics in the form of game theory and coalition theory. Based on this hypothesis, we propose that each particle of PSO will not only follow its own local best solution, but will also follow other superior particles' local best solutions with creditability at the same time. Iteratively, with the creditability declining, the individual will apt to track multiple local best solutions until it is very close the global best solution. In this paper, we present a new type of topologies to emulate this process that extends the basic meaning of standard PSO.

3.2 Algorithm Details

Traditional PSO updates particles' velocities according to evaluations among particles in the whole dimension. However, in practice, some problems show different properties in each dimension. A particle that has high fitness value in some dimensions may have low fitness value due to inferior solutions in other dimensions [7]. Because of that, references [6-7] proposed new strategies to improve the convergence of PSO in different dimensions. In this paper, first, we evaluate other local best solutions in each dimension and find the superior solutions in that dimension for one particle. Then, the velocity of a particle in one dimension is followed by all other superior solutions. The updating equation is expressed as

$$\begin{aligned} v_{i,j}(t+1) = {} & wv_{i,j}(t) + c_1 \sum_{m \in \Psi} r_m c_r^m(t)[p_{m,j} - x_{i,j}(t)] \\ & + c_2 r_g [\tilde{g}_j - x_{i,j}(t)] \end{aligned} \quad (4)$$

where, Ψ is represented as the set where the particles' local best solutions in the jth dimension are equal or smaller than the ith particle's for minimization problem, i.e. $\Psi = \{m | \tilde{f}_{p_m} \le f_{p_i}, m = 1, 2, ..., N\}$, $\tilde{f}_{p_m}$ represents the fitness value when the jth element of the ith local best solution is substituted by the jth element of the mth local best solution; $\tilde{g}_j$ means the jth component of the best solution in the jth dimension for the ith local best solution; the parameters c_1, c_2 and w have the same meaning with equation (1); N is denoted as the population size of the swarm; r_g and r_m $(m = 1, 2, ..., N)$ are also independent random numbers uniformly distributed in the range $[0, 1]$; the credit coefficient $c_r^m(t)$ is given by

$$c_r^m(t) = \begin{cases} \left(1 - \frac{\tilde{f}_{p_m} - \tilde{g}_j}{f_p^1 - \tilde{g}_j}\right)\left(\frac{n_{\max} - t}{n_{\max}}\right) & (f_p^1 - \tilde{g}_j \neq 0) \\ 0 & (f_p^1 - \tilde{g}_j = 0) \end{cases} \quad (5)$$

where f_p^1 is called generalized average fitness of the local best solutions for the particles in set Ψ , i.e. $f_p^1 = \frac{1}{N} \sum_{m \in \Psi} \tilde{f}_{p_m}$; likewise, $n_{\max}$ is the maximum number of iteration. In fact, the key learning strategy GPSO adopts is that each particle tracks multiple local optima in each dimension instead of the whole search space. GPSO can be treated as an extension of the standard PSO by tracking multiple local optima according to the observed social behaviors described in the beginning of this section. Also, from the viewpoint of dimension learning, GPSO can be seen as an extension of CLPSO [7] and CPSO [6]. The pseudocode of GPSO is given in Fig. 1.

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Initialize position X , velocity V and local best P for each particle and
global best G for the whole swarm in d -dimensional space, set the initial
number of iterations t = 0 ;
repeat:
    Calculate inertia weight w(t) according to Eq. (2);
    for each particle i ∈ [1,2,..., M] :
        for each dimension j ∈ [1, 2,..., d] :
            Evaluate the fitness values when the j th element of the i th
            particle's local best solution is substituted by the j th elements
            of other local best solutions and find the set Ψ in equation (4);
            Calculate credit coefficient c_γ^m(t) ;
            Update the velocity v_{i,j}(t+1) and position x_{i,j}(t+1) according
            to Eqs (4), (3);
        endfor
    endfor
    for each particle i ∈ [1,2,..., M] :
        if f(x_i(t+1)) < f(P_i(t+1))
            then P_i(t+1) = x_i(t+1) ;
        if f(P_i(t+1)) < f(G)
            then G = P_i(t+1)
    endfor
until the maximum number of iterations is met.
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Figure 1. The pseudocode of GPSO

4 Numerical Experiments on Benchmarks

4.1 Testing Functions

We chose the following testing functions including a unimodal function, two multimodal functions and a composite functions [6, 7, 9] as the benchmark functions because of their popularity and representative.

1. Rosenbrock's function (RF, unimodal):

$$f_1(\mathbf{x}) = \sum_{i=1}^{d-1} \left(100(x_i^2 - x_{i+1})^2 + (x_i - 1)^2\right) \tag{6}$$

(Domain: 2.048, Threshold: 100)

2. Ackley's function (AF, multimodal):

$$\begin{aligned} f_2(\mathbf{x}) = -20 \exp\left(-0.2\sqrt{\tfrac{1}{d}\sum_{i=1}^{d} x_i^2}\right) \\ -\exp\left(\tfrac{1}{d}\sum_{i=1}^{d}\cos(2\pi x_i)\right) + 20 + e \end{aligned} \tag{7}$$

(Domain: 30, Threshold: 5)

3. Griewanks's function (GF, multimodal):

$$f_3(\mathbf{x}) = \sum_{i=1}^{d} \frac{x_i^2}{4000} - \prod_{i=1}^{d} \cos\left(\frac{x_i}{\sqrt{i}}\right) + 1 \tag{8}$$

(Domain: 600, Threshold: 0.1)

4. Composition function 4(CF4 in reference [9]):

f_4 (CF4) is constructed by ten different benchmark functions that consist of two rotated Ackley's functions, two rotated Rastrigin's functions, two rotated Weierstrass functions, two rotated Griewank's functions, and two sphere functions.

(Domain: 5, Threshold: 500)

where, the thresholds are used to test the stability of PSOs. Aiming for a rigorous evaluation on their capability of handling different problems, unimodal and multimodal functions are tested on thirty dimensions and composite functions are tested on ten dimensions, respectively. The global fitness values are all zeros for all the testing functions.

4.2 Parameters' Configuration of PSOs

In this paper, GPSOs are compared with the standard PSO and several improved PSOs including CLPSO [7], CPSO-H [6], FI-PSO [5], UPSO [4] and FDR-PSO [3] under the above benchmarks. In order to create a fair testing platform, the same basic parameters of PSOs were set. For instances, the population size N was set at 20; both the acceleration coefficients c_1 and c_2 were set to 1.49. The inertia weight w is in the range $[0.4, 0.9]$ and declines linearly in iteration as described in equation (2). The maximum iterations were set at 1×10^4 for solving 30-dimensional unimodal and multimodal problems and 1×10^3 for solving 10-dimensional composite functions, respectively. With the above mentioned parameter settings, all the simulations were run for 20 times continuously and independently. The mean values of convergence results, corresponding stability results are listed in Table I (Note that Con. is an abbreviation of convergence, and Sta. is an abbreviation of stability in the tables). The definition of stability means that the run is successful if it reaches the predefined threshold.

4.3 Comparative Results with Other PSOs

Rosenbrock's function (f_1) is often treated as a multimodal problem. It is very difficult to locate the minimum due to a narrow ridge from the local optima to the global optimum. Encouragingly, GPSO delivers the best performance in convergence. PSO shows the worst stability result. Ackley's function (f_2) is a multimodal function with many local minima positioned on a regular grid [6]. As shown in Table I, GPSO exhibits the best performance in convergence, and CLPSO, CPSO-H, FIPSO and FDR-PSO deliver similar results. PSO only obtains 60% success of rate over all the run times. Griewanks's function (f_3) has undesirable properties as the dimensionality of the function is increased such that the basin of attraction containing the global opti-

Table 1. Results of multimodal functions

Function	F_1		F_2	
	Con.	Sta.	Con.	Sta.
GPSO	5.21e-002	20	7.11e-015	20
CLPSO	1.83e+001	20	1.10e-014	20
CPSO-H	1.99e-001	20	4.65e-014	20
FIPSO	2.21e+001	20	1.76e-014	20
UPSO	8.82e+00	20	1.16e+000	20
FDR-PSO	3.51e-001	20	5.06e-014	20
PSO	1.93e+002	14	5.87e+000	12
Function	F_3		F_4	
	Con.	Sta.	Con.	Sta.
GPSO	0	20	3.36e+002	20
CLPSO	0	20	3.48e+002	20
CPSO-H	1.02e-002	20	5.35e+002	9
FIPSO	1.21e-006	20	3.50e+002	20
UPSO	6.50e-003	20	3.50e+002	19
FDR-PSO	1.69e-002	20	4.59e+002	11
PSO	2.26e+001	16	5.83e+002	8

mum appears to encompass a larger percentage of the total space as the search space grow. GPSO and CLPSO are able to find the theoretical optimum over the run times. Other modified PSOs deliver the similar results in convergence. Composition function 4 (f_4) called a hybrid composition function is composed by ten different benchmark functions [9]. GPSO also performs well in this composition function, while PSO and CPSO-H deliver the worst convergence and stability results. FDR-PSO also shows unstable property for this composition problem. In summary, GPSO consistently performs better than other PSOs, and It exhibits capability of solving multimodal problems based on our experiments.

5 Conclusion

In this paper, a generalized particle swarm optimizer is presented based on the relationships in social behavior. By analyzing some real-world phenomena assigned to evolutionary particles, the updating velocity equations can be treated as generalized models to extend the topology of the original PSO and some existing modified PSOs. Comparative studies confirm that the proposed GPSO is able to deliver marked improvements over the state-of-the-art PSO methods. Apart from the same parameters as the standard PSO, GPSO does not require any other parameter settings, which is another attractive characteristic for evolutionary algorithm. Thus, GPSO is a powerful tool to solve multimodal functions optimization. Despite the encouraging results delivered by the proposed GPSO, we need to do further investigation by developing a more efficient cooperative scheme utilizing multiple local optima to improve the performance of GPSO in different classes of problems.

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